

PHỤ LỤC A: Biến đổi Z của dãy các tín hiệu thường gặp

STT	$x(n), n \geq 0$	$X(z)$	Miền hội tụ
1	$x(0) = k, x(n) = 0, n \geq 1$	k	$ z > 0$
2	$x(m) = k, x(n) = 0, n \neq m$	kz^{-m}	$ z > 0$
3	k	$kz / z - 1$	$ z > 1$
4	nk	$kz / (z - 1)^2$	$ z > 1$
5	n^2k	$kz(z + 1) / (z - 1)^3$	$ z > 1$
6	ke^{-anT}, a là số phức	$kz / (z - e^{-aT})$	$ z > e^{-aT} $
7	kne^{-anT}, a là số phức	$kze^{-aT} / (z - e^{-aT})^2$	$ z > e^{-aT} $
8	$\sin(\omega_0 nT)$	$\frac{z \sin(\omega_0 T)}{z^2 - 2z \cos(\omega_0 T) + 1}$	$ z > 1$
9	$\cos(\omega_0 nT)$	$\frac{z(z - \cos(\omega_0 T))}{z^2 - 2z \cos(\omega_0 T) + 1}$	$ z > 1$
10	$e^{-anT} \sin(\omega_0 nT)$	$\frac{ze^{-aT} \sin(\omega_0 T)}{z^2 - 2ze^{-aT} \cos(\omega_0 T) + e^{-2aT}}$	$ z > e^{-aT}$
11	$e^{-anT} \cos(\omega_0 nT)$	$\frac{ze^{-aT} [ze^{-aT} - \cos(\omega_0 T)]}{z^2 - 2ze^{-aT} \cos(\omega_0 T) + e^{-2aT}}$	$ z > e^{-aT}$
12	a^n	$z / (z - a)$	$ z > a$
13	na^n	$az / (z - a)^2$	$ z > a$
14	$n^2 a^n$	$az(z + a) / (z - a)^3$	$ z > a$
15	$\sinh(\omega_0 nT)$	$\frac{z \sinh(\omega_0 T)}{z^2 - 2z \cosh(\omega_0 T) + 1}$	$ z > \cosh(\omega_0 T)$
16	$\cosh(\omega_0 nT)$	$\frac{z[z - \cosh(\omega_0 T)]}{z^2 - 2z \cosh(\omega_0 T) + 1}$	$ z > \sinh(\omega_0 T)$
17	$a^n / n!$	$e^{a/z}$	$ z > 0$
18	$[\ln a]^n / n!$	$a^{1/z}$	$ z > 0$

PHỤ LỤC B

Bảng tóm tắt các tính chất cơ bản của phép biến đổi Fourier

$$\widehat{X}(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft} x(t) dt$$

Tính chất	Hàm $x(t)$	Biến đổi Fourier $\widehat{X}(f)$
1. Tuyến tính	$Ax_1(t) + Bx_2(t)$	$A\widehat{X}_1(f) + B\widehat{X}_2(f)$
2. Đồng dạng	$x(at)$	$\frac{1}{ a } \widehat{X}(f/a)$
3. Liên hợp	$\overline{x(t)}$	$\overline{\widehat{X}(-f)}$
4. Đối ngẫu	$\widehat{X}(t)$	$x(-f)$
5. Trễ	$x(t - T_d)$	$e^{-i2\pi T_d f} \widehat{X}(f)$
6. Dịch chuyển ảnh	$e^{i2\pi f_0 t} x(t)$	$\widehat{X}(f - f_0)$
7. Điều chế	$x(t) \cos 2\pi f_0 t$	$\frac{1}{2} \widehat{X}(f - f_0) + \frac{1}{2} \widehat{X}(f + f_0)$
8. Đạo hàm	$\frac{d^n x(t)}{dt^n}$	$(i2\pi f)^n \widehat{X}(f)$
9. Tích phân	$\int_{-\infty}^t x(u) du$	$\frac{1}{i2\pi f} \widehat{X}(f) + \frac{1}{2} \widehat{X}(0) \delta(f)$
10. Đạo hàm ảnh	$t^n x(t)$	$(-i2\pi)^{-n} \frac{d^n \widehat{X}(f)}{df^n}$
11. Tích chập	$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(u) * x_2(t - u) du$	$\widehat{X}_1(f) \widehat{X}_2(f)$
12. Tích	$x_1(t) x_2(t)$	$\widehat{X}_1(f) * \widehat{X}_2(f)$

PHỤ LỤC C:
Các cặp biến đổi Fourier thường gặp

STT	Hàm $x(t)$	Biến đổi Fourier $\widehat{X}(f)$
1	$\Pi(t / T)$	$T \operatorname{sinc}(Tf)$
2	$2W \operatorname{sinc}(2Wt)$	$\Pi(f / 2W)$
3	$\Lambda(t / T)$	$T \operatorname{sinc}^2(Tf)$
4	$e^{-\lambda t} u(t); \lambda > 0$	$\frac{1}{\lambda + i2\pi f}$
5	$te^{-\lambda t} u(t); \lambda > 0$	$\frac{1}{(\lambda + i2\pi f)^2}$
6	$e^{-\lambda t }; \lambda > 0$	$\frac{2\lambda}{\lambda^2 + (2\pi f)^2}$
7	$\frac{1}{\lambda^2 + (2\pi t)^2}; \lambda > 0$	$\frac{e^{-\lambda f }}{2\lambda}$
8	$\delta(t)$	1
9	1	$\delta(f)$
10	$\delta(t - t_0)$	$e^{-i2\pi f t_0}$
11	$e^{i2\pi f_0 t}$	$\delta(f - f_0)$
12	$\cos 2\pi f_0 t$	$\frac{\delta(f - f_0) + \delta(f + f_0)}{2}$
13	$\sin 2\pi f_0 t$	$\frac{\delta(f - f_0) - \delta(f + f_0)}{2i}$
14	$u(t)$	$\frac{1}{i2\pi f} + \frac{1}{2}\delta(f)$
15	$\operatorname{sgn}(t)$	$\frac{1}{i\pi f}$
16	$\frac{1}{\pi t}$	$-i \operatorname{sgn}(f)$

PHỤ LỤC D

Bảng tóm tắt các tính chất cơ bản của phép biến đổi Laplace

$$X(s) = \int_0^{\infty} e^{-st} x(t) dt$$

Tính chất	Hàm $x(t)$	Biến đổi Laplace $X(s)$
1. Tuyến tính	$Ax_1(t) + Bx_2(t)$	$AX_1(s) + BX_2(s)$
2. Đồng dạng	$x(at)$	$\frac{1}{a} X\left(\frac{s}{a}\right)$
3. Dịch chuyển ảnh	$e^{at} x(t)$	$X(s - a)$
4. Trễ	$x(t - a)\eta(t - a)$	$e^{-as} X(s)$
5. Đạo hàm	$\frac{dx(t)}{dt}$	$sX(s) - x(0)$
6. Đạo hàm	$\frac{d^n x(t)}{dt^n}$	$s^n X(s) - s^{n-1}x(0) - \dots - x^{(n-1)}(0)$
7. Đạo hàm ảnh	$t^n x(t)$	$(-1)^n \frac{d^n X(s)}{ds^n}$
8. Tích phân	$\int_0^t x(u) du$	$\frac{X(s)}{s}$
9. Tích phân	$\int_0^t \dots \int_0^t x(u) du^n = \int_0^t \frac{(t-u)^{n-1}}{(n-1)!} x(u) du$	$\frac{X(s)}{s^n}$
10. Tích phân ảnh	$\frac{x(t)}{t}$	$\int_s^{\infty} X(u) du$
11. Tích chập	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
12. Duhamel	$x_1(0)x_2(t) + x_1' * x_2(t)$	$sX_1(s)X_2(s)$

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13. Tuần hoàn	$x(t + T) = x(t)$	$X(s) = \frac{\int_0^T e^{-st} x(t) dt}{1 - e^{-sT}}$
14.	$\frac{1}{\sqrt{\pi t}} \int_0^\infty e^{-\frac{u^2}{4t}} x(u) du$	$\frac{X(\sqrt{s})}{\sqrt{s}}$
15.	$\int_0^\infty J_0(2\sqrt{ut}) x(u) du$	$\frac{1}{s} X\left(\frac{1}{s}\right)$
16.	$t^{\frac{n}{2}} \int_0^\infty u^{-\frac{n}{2}} J_n(2\sqrt{ut}) x(u) du$	$\frac{1}{s^{n+1}} X\left(\frac{1}{s}\right)$
17.	$\int_0^t J_0(2\sqrt{u(t-u)}) x(u) du$	$\frac{1}{s^2 + 1} X\left(s + \frac{1}{s}\right)$
18.	$x(t^2)$	$\frac{1}{2\sqrt{\pi}} \int_0^\infty u^{-\frac{3}{2}} e^{-\frac{s^2}{4u}} X(u) du$
19.	$\int_0^\infty \frac{t^u x(u)}{\Gamma(u+1)} du$	$\frac{X(\ln s)}{s \ln s}$
20.	$\sum_{k=1}^n \frac{P(a_k)}{Q'(a_k)} e^{a_k t}$	$\frac{P(s)}{Q(s)}$ Bậc $P(s) <$ bậc $Q(s)$, $Q(s)$ chỉ có các nghiệm đơn là a_1, \dots, a_n và không phải là nghiệm của $P(s)$

PHỤ LỤC E

Biến đổi Laplace của các hàm thường gặp

$$X(s) = \int_0^{\infty} e^{-st} x(t) dt$$

TT	Ảnh biến đổi Laplace $X(s)$	Hàm gốc $x(t)$
1.	$\frac{1}{s}$	1
2.	$\frac{1}{s^n}; n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!}$
3.	$\frac{1}{s^\alpha}; \alpha > 0$	$\frac{t^{\alpha-1}}{\Gamma(\alpha)}$
4.	$\frac{1}{s-a}$	e^{at}
5.	$\frac{1}{(s-a)^n}; n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!} e^{at}$
6.	$\frac{1}{(s-a)^\alpha}; \alpha > 0$	$\frac{t^{\alpha-1}}{\Gamma(\alpha)} e^{at}$
7.	$\frac{1}{s^2 + a^2}$	$\frac{\sin at}{a}$
8.	$\frac{s}{s^2 + a^2}$	$\cos at$
9.	$\frac{1}{(s-b)^2 + a^2}$	$\frac{e^{bt} \sin at}{a}$
10.	$\frac{s-b}{(s-b)^2 + a^2}$	$e^{bt} \cos at$
11.	$\frac{1}{s^2 - a^2}$	$\frac{\sinh at}{a}$
12.	$\frac{s}{s^2 - a^2}$	$\cosh at$

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13.	$\frac{1}{(s-b)^2 - a^2}$	$\frac{e^{bt} \sinh at}{a}$
14.	$\frac{s-b}{(s-b)^2 - a^2}$	$e^{bt} \cosh at$
15.	$\frac{1}{(s^2 + a^2)^2}$	$\frac{\sin at - at \cos at}{2a^3}$
16.	$\frac{s}{(s^2 + a^2)^2}$	$\frac{t \sin at}{2a}$
17.	$\frac{s^2}{(s^2 + a^2)^2}$	$\frac{\sin at + at \cos at}{2a}$
18.	$\frac{s^3}{(s^2 + a^2)^2}$	$\cos at - \frac{1}{2} at \sin at$
19.	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$	$t \cos at$
20.	$\frac{1}{(s^2 - a^2)^2}$	$\frac{at \cosh at - \sinh at}{2a^3}$
21.	$\frac{s}{(s^2 - a^2)^2}$	$\frac{t \sinh at}{2a}$
22.	$\frac{s^2}{(s^2 - a^2)^2}$	$\frac{\sinh at + at \cosh at}{2a}$
23.	$\frac{s^3}{(s^2 - a^2)^2}$	$\cosh at + \frac{1}{2} at \sinh at$
24.	$\frac{s^2 + a^2}{(s^2 - a^2)^2}$	$t \cosh at$

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25.	$\frac{1}{(s^2 + a^2)^3}$	$\frac{(3 - a^2 t^2) \sin at - 3at \cos at}{8a^5}$
26.	$\frac{s}{(s^2 + a^2)^3}$	$\frac{t \sin at - at^2 \cos at}{8a^3}$
27.	$\frac{s^2}{(s^2 + a^2)^3}$	$\frac{(1 + a^2 t^2) \sin at - at \cos at}{8a^3}$
28.	$\frac{s^3}{(s^2 + a^2)^3}$	$\frac{3t \sin at + at^2 \cos at}{8a}$
29.	$\frac{s^4}{(s^2 + a^2)^3}$	$\frac{(3 - a^2 t^2) \sin at + 5at \cos at}{8a}$
30.	$\frac{s^5}{(s^2 + a^2)^3}$	$\frac{(8 - a^2 t^2) \cos at - 7at \sin at}{8}$
31.	$\frac{3s^2 - a^2}{(s^2 + a^2)^3}$	$\frac{t^2 \sin at}{2a}$
32.	$\frac{s^3 - 3a^2 s}{(s^2 + a^2)^3}$	$\frac{1}{2} t^2 \cos at$
33.	$\frac{s^4 - 6a^2 s^2 + a^4}{(s^2 + a^2)^4}$	$\frac{1}{6} t^3 \cos at$
34.	$\frac{s^3 - a^2 s}{(s^2 + a^2)^4}$	$\frac{t^3 \sin at}{24a}$
35.	$\frac{1}{(s^2 - a^2)^3}$	$\frac{(3 + a^2 t^2) \sinh at - 3at \cosh at}{8a^5}$
36.	$\frac{s}{(s^2 - a^2)^3}$	$\frac{at^2 \cosh at - t \sinh at}{8a^3}$

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37.	$\frac{s^2}{(s^2 - a^2)^3}$	$\frac{at \cosh at + (a^2 t^2 - 1) \sinh at}{8a^3}$
38.	$\frac{s^3}{(s^2 - a^2)^3}$	$\frac{3t \sinh at + at^2 \cosh at}{8a}$
39.	$\frac{s^4}{(s^2 - a^2)^3}$	$\frac{(3 + a^2 t^2) \sinh at + 5at \cosh at}{8a}$
40.	$\frac{s^5}{(s^2 - a^2)^3}$	$\frac{(8 + a^2 t^2) \cosh at + 7at \sinh at}{8}$
41.	$\frac{3s^2 + a^2}{(s^2 - a^2)^3}$	$\frac{t^2 \sinh at}{2a}$
42.	$\frac{s^3 + 3a^2 s}{(s^2 - a^2)^3}$	$\frac{1}{2} t^2 \cosh at$
43.	$\frac{s^4 + 6a^2 s^2 + a^4}{(s^2 - a^2)^4}$	$\frac{1}{6} t^3 \cosh at$
44.	$\frac{s^3 + a^2 s}{(s^2 - a^2)^4}$	$\frac{t^3 \sinh at}{24a}$
45.	$\frac{1}{s^3 + a^3}$	$\frac{e^{at/2}}{3a^2} \left\{ \sqrt{3} \sin \frac{\sqrt{3} at}{2} - \cos \frac{\sqrt{3} at}{2} + e^{-3at/2} \right\}$
46.	$\frac{1}{s^3 + a^3}$	$\frac{e^{at/2}}{3a} \left\{ \sqrt{3} \sin \frac{\sqrt{3} at}{2} + \cos \frac{\sqrt{3} at}{2} - e^{-3at/2} \right\}$
47.	$\frac{s^2}{s^3 + a^3}$	$\frac{1}{3} \left(e^{-at} + 2e^{at/2} \cos \frac{\sqrt{3} at}{2} \right)$

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48.	$\frac{1}{s^3 - a^3}$	$\frac{e^{-at/2}}{3a^2} \left\{ e^{3at/2} - \sqrt{3} \sin \frac{\sqrt{3} at}{2} - \cos \frac{\sqrt{3} at}{2} \right\}$
49.	$\frac{1}{s^3 - a^3}$	$\frac{e^{-at/2}}{3a} \left\{ \sqrt{3} \sin \frac{\sqrt{3} at}{2} - \cos \frac{\sqrt{3} at}{2} + e^{3at/2} \right\}$
50.	$\frac{s^2}{s^3 - a^3}$	$\frac{1}{3} \left(e^{at} + 2e^{-at/2} \cos \frac{\sqrt{3} at}{2} \right)$
51.	$\frac{1}{s^4 + 4a^4}$	$\frac{1}{4a^3} \left\{ \sin at \cosh at - \cos at \sinh at \right\}$
52.	$\frac{s}{s^4 + 4a^4}$	$\frac{\sin at \sinh at}{2a^2}$
53.	$\frac{s^2}{s^4 + 4a^4}$	$\frac{1}{2a} \left\{ \sin at \cosh at + \cos at \sinh at \right\}$
54.	$\frac{s^3}{s^4 + 4a^4}$	$\cos at \cosh at$
55.	$\frac{1}{s^4 - a^4}$	$\frac{1}{2a^3} \left\{ \sinh at - \sin at \right\}$
56.	$\frac{s}{s^4 - a^4}$	$\frac{1}{2a^2} \left\{ \cosh at - \cos at \right\}$
57.	$\frac{s^2}{s^4 - a^4}$	$\frac{1}{2a} \left\{ \sinh at + \sin at \right\}$
58.	$\frac{s^3}{s^4 - a^4}$	$\frac{1}{2a} \left\{ \cosh at + \cos at \right\}$
59.	$\frac{1}{\sqrt{s+a} + \sqrt{s+b}}$	$\frac{e^{-bt} - e^{-at}}{2(b-a)\sqrt{\pi t^3}}$
60.	$\frac{1}{s\sqrt{s+a}}$	$\frac{\operatorname{erf} \sqrt{at}}{\sqrt{a}}$

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61.	$\frac{1}{\sqrt{s(s-a)}}$	$\frac{e^{at} \operatorname{erf} \sqrt{at}}{\sqrt{a}}$
62.	$\frac{1}{\sqrt{s-a+b}}$	$e^{at} \left\{ \frac{1}{\sqrt{\pi t}} - be^{b^2 t} \operatorname{erfc}(b\sqrt{t}) \right\}$
63.	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
64.	$\frac{1}{\sqrt{s^2-a^2}}$	$I_0(at)$
65.	$\frac{(\sqrt{s^2+a^2}-s)^n}{\sqrt{s^2+a^2}}; n > -1$	$a^n J_n(at)$
66.	$\frac{(s-\sqrt{s^2-a^2})^n}{\sqrt{s^2-a^2}}; n > -1$	$a^n I_n(at)$
67.	$\frac{e^{b(s-\sqrt{s^2+a^2})}}{\sqrt{s^2+a^2}}$	$J_0(a\sqrt{t(t+2b)})$
68.	$\frac{e^{-b\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$\eta(t-b)J_0(a\sqrt{t^2-b^2})$
69.	$\frac{1}{\sqrt{(s^2+a^2)^3}}$	$\frac{tJ_1(at)}{a}$
70.	$\frac{s}{\sqrt{(s^2+a^2)^3}}$	$tJ_0(at)$
71.	$\frac{s^2}{\sqrt{(s^2+a^2)^3}}$	$J_0(at) - tJ_1(at)$
72.	$\frac{1}{s(e^s-1)} = \frac{e^{-s}}{s(1-e^{-s})}$	$x(t) = n, n \leq t < n+1, n = 0, 1, 2, \dots$
73.	$\frac{1}{s(e^s-r)} = \frac{e^{-s}}{s(1-re^{-s})}$	$x(t) = \sum_{k=1}^{[t]} r^k; [t]$ là phần nguyên của t

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74.	$\frac{e^s - 1}{s(e^s - r)} = \frac{1 - e^{-s}}{s(1 - re^{-s})}$	$x(t) = r^n, n \leq t < n + 1, n = 0, 1, 2, \dots$
75.	$\frac{e^{-s/a}}{\sqrt{s}}$	$\frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$
76.	$\frac{e^{-s/a}}{\sqrt{s^3}}$	$\frac{\sin 2\sqrt{at}}{\sqrt{\pi a}}$
77.	$\frac{e^{-s/a}}{s^{\alpha+1}}; \alpha > -1$	$\left(\frac{t}{a}\right)^{\alpha/2} J_{\alpha}(2\sqrt{at})$
78.	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}} e^{-\frac{a^2}{4t}}$
79.	$e^{-a\sqrt{s}}$	$\frac{a}{2\sqrt{\pi t^3}} e^{-\frac{a^2}{4t}}$
80.	$\frac{1 - e^{-a\sqrt{s}}}{s}$	$\operatorname{erf}\left(\frac{a}{2\sqrt{t}}\right)$
81.	$\frac{e^{-a\sqrt{s}}}{s}$	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$
82.	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$	$e^{b(bt+a)} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$
83.	$\frac{e^{-a/\sqrt{s}}}{s^{\alpha+1}}; \alpha > -1$	$\frac{1}{\sqrt{\pi t} a^{2\alpha+1}} \int_0^{\infty} u^{\alpha} e^{-\frac{u^2}{4a^2 t}} J_{2\alpha}(2\sqrt{u}) du$
84.	$\ln\left(\frac{s+a}{s+b}\right)$	$\frac{e^{-bt} - e^{-at}}{t}$
85.	$\frac{1}{2s} \ln\left(\frac{s^2 + a^2}{a^2}\right)$	$\operatorname{Ci}(at)$
86.	$\frac{1}{s} \ln\left(\frac{s+a}{a}\right)$	$\operatorname{Ei}(at)$

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87.	$-\frac{\gamma + \ln s}{s}$	$\ln t$; γ là hằng số Euler
88.	$\ln \left(\frac{s^2 + a^2}{s^2 + b^2} \right)$	$\frac{2(\cos bt - \cos at)}{t}$
89.	$\frac{\pi^2}{6s} + \frac{(\gamma + \ln s)^2}{s}$	$\ln^2 t$; γ là hằng số Euler
90.	$\frac{\ln s}{s}$	$-(\ln t + \gamma)$
91.	$\frac{\ln^2 s}{s}$	$(\ln t + \gamma)^2 - \frac{\pi^2}{6}$
92.	$\frac{\Gamma(\alpha + 1) - \Gamma(\alpha + 1)s}{s^{\alpha+1}} ; \alpha > -1$	$t^\alpha \ln t$
93.	$\arctan \left(\frac{a}{s} \right)$	$\frac{\sin at}{t}$
94.	$\frac{1}{s} \arctan \left(\frac{a}{s} \right)$	$\text{Si}(at)$
95.	$\frac{e^{a/s}}{\sqrt{s}} \text{erfc} \left(\sqrt{a/s} \right)$	$\frac{e^{-2\sqrt{at}}}{\sqrt{\pi t}}$
96.	$e^{s^2/4a^2} \text{erfc}(s/2a)$	$\frac{2a}{\sqrt{\pi}} e^{-a^2 t^2}$
97.	$\frac{e^{s^2/4a^2} \text{erfc}(s/2a)}{s}$	$\text{erf}(at)$
98.	$\frac{e^{as}}{\sqrt{s}} \text{erfc}(\sqrt{as})$	$\frac{1}{\sqrt{\pi(t+a)}}$
99.	$e^{as} \text{Ei}(as)$	$\frac{1}{t+a}$
100.	$\frac{\cos as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} - \sin as \text{Ci}(as)}{a}$	$\frac{1}{t^2 + a^2}$

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101.	$\sin as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} + \cos as \text{Ci}(as)$	$\frac{t}{t^2 + a^2}$
102.	$\frac{\cos as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} - \sin as \text{Ci}(as)}{s}$	$\arctan(t/a)$
103.	$\frac{\sin as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} + \cos as \text{Ci}(as)}{s}$	$\frac{1}{2} \ln \left(\frac{t^2 + a^2}{a^2} \right)$
104.	$\left\{ \frac{\pi}{2} - \text{Si}(as) \right\}^2 + \text{Ci}^2(as)$	$\frac{1}{t} \ln \left(\frac{t^2 + a^2}{a^2} \right)$
105.	1	$\delta(t)$ - hàm Dirac
106.	e^{-as}	$\delta(t - a)$
107.	$\frac{e^{-as}}{s}$	$\eta(t - a)$
108.	$\frac{1 \sinh xs}{s \sinh as}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{a} \cos \frac{n\pi t}{a}$
109.	$\frac{1 \sinh xs}{s \cosh as}$	$\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \sin \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
110.	$\frac{1 \cosh xs}{s \sinh as}$	$\frac{t}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos \frac{n\pi x}{a} \sin \frac{n\pi t}{a}$
111.	$\frac{1 \cosh xs}{s \cosh as}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \cos \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
112.	$\frac{1 \sinh xs}{s^2 \sinh as}$	$\frac{xt}{a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin \frac{n\pi x}{a} \cos \frac{n\pi t}{a}$
113.	$\frac{1 \sinh xs}{s^2 \cosh as}$	$x + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \sin \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$

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114.	$\frac{1}{s^2} \frac{\cosh xs}{\sinh as}$	$\frac{t^2}{2a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{a} \left(1 - \cos \frac{n\pi t}{a} \right)$
115.	$\frac{1}{s^2} \frac{\cosh xs}{\cosh as}$	$t + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
116.	$\frac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{2\pi}{a^2} \sum_{n=1}^{\infty} (-1)^n n e^{-n^2\pi^2 t/a^2} \sin \frac{n\pi x}{a}$
117.	$\frac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\frac{\pi}{a^2} \sum_{n=1}^{\infty} (-1)^{n-1} (2n-1) e^{-\frac{(2n-1)^2\pi^2 t}{4a^2}} \cos \frac{(2n-1)\pi x}{2a}$
118.	$\frac{1}{\sqrt{s}} \frac{\sinh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n-1} e^{-\frac{(2n-1)^2\pi^2 t}{4a^2}} \sin \frac{(2n-1)\pi x}{2a}$
119.	$\frac{1}{\sqrt{s}} \frac{\cosh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 t}{a^2}} \cos \frac{n\pi x}{2a}$
120.	$\frac{1}{s} \frac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-\frac{n^2\pi^2 t}{a^2}} \sin \frac{n\pi x}{2a}$
121.	$\frac{1}{s} \frac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} e^{-\frac{(2n-1)^2\pi^2 t}{4a^2}} \cos \frac{(2n-1)\pi x}{2a}$
122.	$\frac{1}{s^2} \frac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{xt}{a} + \frac{2a^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (1 - e^{-\frac{n^2\pi^2 t}{a^2}}) \sin \frac{n\pi x}{2a}$
123.	$\frac{1}{s^2} \frac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\frac{x^2 - a^2}{2} + t - \frac{16a^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} e^{-\frac{(2n-1)^2\pi^2 t}{4a^2}} \cos \frac{(2n-1)\pi x}{2a}$
124.	$\frac{1}{s} \frac{J_0(ix\sqrt{s})}{J_0(ia\sqrt{s})}$	$1 - 2 \sum_{n=1}^{\infty} \frac{e^{-\lambda_n^2 t/a^2} J_0(\lambda_n x/a)}{\lambda_n J_1(\lambda_n)}$ $\lambda_1, \lambda_2, \dots$ là các nghiệm dương của $J_0(\lambda) = 0$

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125.	$\frac{1}{s^2} \frac{J_0(ix\sqrt{s})}{J_0(ia\sqrt{s})}$	$\frac{x^2 - a^2}{4} + t + 2a^2 \sum_{n=1}^{\infty} \frac{e^{-\lambda_n^2 t/a^2} J_0(\lambda_n x/a)}{\lambda_n^3 J_1(\lambda_n)}$ $\lambda_1, \lambda_2, \dots$ là các nghiệm dương của $J_0(\lambda) = 0$
126.	$\frac{1}{as^2} \tanh\left(\frac{as}{2}\right)$	
127.	$\frac{1}{s} \tanh\left(\frac{as}{2}\right)$	
128.	$\frac{\pi a}{a^2 s^2 + \pi^2} \cosh\left(\frac{as}{2}\right)$	
129.	$\frac{\pi a}{(a^2 s^2 + \pi^2)(1 - e^{-as})}$	
130.	$\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$	
131.	$\frac{e^{-as}}{s} (1 - e^{-bs})$	$\eta(t-a) - \eta(t-a-b)$
132.	$\frac{1}{s(1 - e^{-as})}$	$\sum_{n=1}^{\infty} n [\eta(t - (n-1)a) - \eta(t - na)]$
133.	$\frac{e^{-s} + e^{-2s}}{s(1 - e^{-s})^2}$	$\sum_{n=0}^{\infty} n^2 [\eta(t-n) - \eta(t-(n+1))]$
134.	$\frac{1 - e^{-s}}{s(1 - re^{-as})^2}$	$\sum_{n=0}^{\infty} r^n [\eta(t-n) - \eta(t-(n+1))]$
135.	$\frac{\pi a(1 + e^{-as})}{a^2 s^2 + \pi^2}$	$(\eta(t) - \eta(t-a)) \sin \frac{\pi t}{a}$

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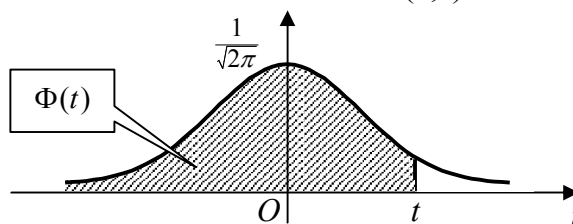
PHỤ LỤC F

GIÁ TRỊ HÀM MẬT ĐỘ XÁC SUẤT PHÂN BỐ CHUẨN TẮC $\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$

<i>t</i>	0	1	2	3	4	5	6	7	8	9
0,0	0,3989	3989	3989	3988	3986	3984	3982	3980	3977	3973
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0,2	3910	3902	3894	3885	3876	3867	3857	3847	3836	3825
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0,4	3683	3668	3653	3637	3621	3605	3589	3572	3555	3538
0,5	3521	3503	3485	3467	3448	3429	3410	3391	3372	3352
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0,7	3123	3101	3079	3056	3034	3011	2989	2966	2943	2920
0,8	2897	2874	2850	2827	2803	2780	2756	2732	2709	2685
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1,0	0,2420	2396	2370	2347	2320	2299	2275	2251	2227	2203
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1,2	1942	1919	1895	1872	1849	1826	1804	1781	1758	1736
1,3	1714	1691	1669	1647	1626	1604	1582	1561	1539	1518
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973	0957
1,7	0940	0925	0909	0893	0878	0863	0848	0833	0818	0804
1,8	0790	0775	0761	0748	0734	0721	0707	0694	0681	0669
1,9	0656	0644	0632	0620	0608	0596	0584	0573	0562	0551
2,0	0,0540	0529	0519	0508	0498	0488	0478	0468	0459	0449
2,1	0440	0431	0422	0413	0404	0396	0387	0379	0371	0363
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2,3	0283	0277	0270	0264	0258	0252	0246	0241	0235	0229
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0139
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2,7	0104	0101	0099	0096	0093	0091	0088	0086	0084	0081
2,8	0079	0077	0075	0073	0071	0069	0067	0065	0063	0061
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3,0	0,0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3,1	0033	0032	0031	0030	0029	0028	0027	0026	0025	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3,4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3,5	0009	0008	0008	00080	0008	0007	0007	0007	0007	0006
3,6	0005	0005	0005	0005	0005	0005	0005	0005	0005	0004
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3,8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

GIÁ TRỊ HÀM PHÂN BỐ CHUẨN TẮC N(0;1)

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{x^2}{2}} dx$$



<i>t</i>	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0,1	5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0,2	5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0,3	6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0,4	6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0,5	0,6915	6950	6985	7019	7054	7088	7123	7156	7190	7224
0,6	7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0,7	7580	7611	7642	7673	7703	7734	7764	7794	7823	7852
0,8	7881	7910	7939	7967	7995	8023	8051	8078	8106	8132
0,9	8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1,0	0,8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1,1	8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1,2	8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1,3	9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1,4	9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1,5	0,9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1,6	9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1,7	9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1,8	9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1,9	9712	9719	9726	9732	9738	9744	9750	9756	9761	9767
2,0	0,9773	9778	9783	9788	9793	9798	9803	9808	9812	9817
2,1	9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2,2	9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2,3	9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2,4	9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2,5	0,9938	9940	9941	9943	9945	9946	9948	9949	9951	9952
2,6	9953	9955	9956	9957	9959	9960	9961	9962	9963	9964
2,7	9965	9966	9967	9968	9969	9970	9971	9972	9973	9974
2,8	9974	9975	9976	9977	9977	9978	9979	9979	9980	9981
2,9	9981	9982	9982	9983	9984	9984	9985	9985	9986	9986
<i>t</i>	3,0	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9
$\Phi(t)$	0,9987	9990	9993	9995	9996	9997	9998	9999	9999	9999